

PROOF OF FORMULA 4.229.4

$$\int_0^1 \ln(\ln 1/x) (\ln 1/x)^{\mu-1} dx = \psi(\mu)\Gamma(\mu)$$

The change of variables $t = \ln 1/x = -\ln x$ gives

$$\int_0^1 \ln(\ln 1/x) (\ln 1/x)^{\mu-1} dx = \int_0^\infty e^{-t} t^{\mu-1} \ln t dt.$$

The result now follows from differentiating the expression

$$\Gamma(\mu) = \int_0^\infty t^{\mu-1} e^{-t} dt$$

with respect to the parameter μ and using $\psi(\mu) = \Gamma'(\mu)/\Gamma(\mu)$.