PROOF OF FORMULA 4.229.6

$$\int_0^1 \ln(a - \ln x)\, dx = \ln a - e^a \mathrm{Ei}(-a)$$

Let $t = a - \ln x$ to obtain

$$\int_0^1 \ln(a - \ln x)\, dx = e^a \int_a^\infty e^{-t} \ln t\, dt.$$

Integration by parts gives

$$\int_0^1 \ln(a - \ln x)\, dx = e^a \left[e^{-a} \ln a + \int_a^\infty \frac{e^t}{t}\, dt \right].$$

The result follows by the change of variables $t = -s$.