PROOF OF FORMULA 4.231.1

$$\int_0^1 \frac{\ln x \, dx}{1+x} = -\frac{\pi^2}{12}$$

Let $t = -\ln x$ to obtain

$$\int_0^1 \frac{\ln x \, dx}{1+x} = -\int_0^\infty \frac{t \, dt}{1+e^t}.$$

Use the identity

$$\frac{1}{1+e^t} = \frac{e^{-t}}{1+e^{-t}} = \sum_{k=0}^\infty (-1)^k e^{-(k+1)t} = -\sum_{k=1}^\infty (-1)^k e^{-kt}$$

to obtain

$$\int_0^1 \frac{\ln x \, dx}{1+x} = \sum_{k=1}^\infty (-1)^k \int_0^\infty t e^{-kt} \, dt.$$

The value

$$\int_0^\infty t e^{-kt} \, dt = \frac{1}{k^2}$$

is obtained by integration by parts. Therefore

$$\int_0^1 \frac{\ln x \, dx}{1+x} = \sum_{k=1}^\infty \frac{(-1)^k}{k^2} = -\frac{\pi^2}{12}.$$