PROOF OF FORMULA 4.231.13

$$\int_0^1 \frac{\ln x\,dx}{1-x^2} = -\frac{\pi^2}{8}$$

The partial fraction decomposition

$$\frac{1}{1-x^2} = \frac{1}{2}\left(\frac{1}{1+x} + \frac{1}{1-x}\right)$$

gives

$$\int_0^1 \frac{\ln x\,dx}{1-x^2} = \frac{1}{2}\int_0^1 \frac{\ln x\,dx}{1+x} + \frac{1}{2}\int_0^1 \frac{\ln x\,dx}{1-x}.$$

The values

$$\int_0^1 \frac{\ln x\,dx}{1+x} = -\frac{\pi^2}{12},$$

and

$$\int_0^1 \frac{\ln x\,dx}{1-x} = -\frac{\pi^2}{6},$$

given in **4.231.1** and **4.231.2**, respectively, give the result.