PROOF OF FORMULA 4.231.3

$$\int_0^1 \frac{x \, \ln x \, dx}{1-x} = 1 - \frac{\pi^2}{6}$$

The partial fraction decomposition

$$\frac{x}{1-x} = \frac{1}{1-x} - 1,$$

shows that

$$\int_0^1 \frac{x \, \ln x \, dx}{1-x} = \int_0^1 \frac{\ln x \, dx}{1-x} - \int_0^1 \ln x \, dx.$$

Formula **4.231.2** states that the first integral is $-\pi^2/6$. Integration by parts, shows that the second integral is -1.