PROOF OF FORMULA 4.234.4

$$\int_0^\infty \frac{1-x^2}{(1+x^2)^2}\,\ln x\,dx = -\frac{\pi}{2}$$

Observe that

$$\frac{d}{dx}\frac{x}{1+x^2} = \frac{1-x^2}{(1+x^2)^2}$$

and integrating by parts produces

$$\int_0^\infty \frac{1-x^2}{(1+x^2)^2}\,\ln x\,dx = -\int_0^\infty \frac{dx}{1+x^2} = -\frac{\pi}{2}.$$