PROOF OF FORMULA 4.241.10

$$\int_0^1 x\sqrt{1-x^2}\,\ln x\,dx = \frac{1}{9}(3\ln 2 - 4)$$

Let $t = x^2$ to obtain

$$\int_0^1 x\sqrt{1-x^2}\,\ln x\,dx = \frac{1}{4}\int_0^1 \sqrt{1-t}\,\ln t\,dt.$$

The change of variables $t = e^{-u}$ now gives

$$\int_0^1 \sqrt{1-t}\,\ln t\,dt = -\int_0^\infty ue^{-u}\sqrt{1-e^{-u}}\,du.$$

This last integral was evaluated in **3.451.1** and its value is $4(4-3\ln 2)/9$. This gives the result.