PROOF OF FORMULA 4.247.1

$$\int_0^1 \frac{\ln x\,dx}{\sqrt[n]{1-x^{2n}}} = -\frac{\pi}{8n^2\sin(\pi/2n)} B\left(\frac{1}{2n}, \frac{1}{2n}\right).$$

The change of variable $t = x^{2n}$ gives

$$\int_0^1 \frac{\ln x\,dx}{\sqrt[n]{1-x^{2n}}} = \frac{1}{4n^2}\int_0^1 t^{1/2n-1}(1-t)^{-1/n}\,\ln t\,dt.$$

In the proof of formula **4.253.1**. it was shown that

$$\int_0^1 t^{a-1}(1-t)^{b-1}\ln t\,dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\left[\psi(a)-\psi(a+b)\right].$$

Therefore

$$\int_0^1 \frac{\ln x\,dx}{\sqrt[n]{1-x^{2n}}} = \frac{\Gamma(1/2n)\Gamma(1-1/n)}{4n^2\Gamma(1-1/2n)}\left[\psi(\tfrac{1}{2n})-\psi(1-\tfrac{1}{2n})\right].$$

The relations

$$\begin{aligned} \Gamma\left(1-\tfrac{1}{n}\right)\Gamma\left(\tfrac{1}{n}\right) &= \pi/\sin(\pi/n) \\ \psi\left(\tfrac{1}{2n}\right)-\psi\left(1-\tfrac{1}{2n}\right) &= -\pi\cot(\pi/2n) \end{aligned}$$

give the result.