

**PROOF OF FORMULA 4.256**

$$\int_0^1 \ln\left(\frac{1}{x}\right) \frac{x^{\mu-1} dx}{\sqrt[n]{(1-x^n)^{n-m}}} = \frac{1}{n^2} B\left(\frac{\mu}{n}, \frac{m}{n}\right) \left[ \psi\left(\frac{\mu+m}{n}\right) - \psi\left(\frac{\mu}{n}\right) \right]$$

The change of variable  $t = x^n$  yields

$$\int_0^1 \ln\left(\frac{1}{x}\right) \frac{x^{\mu-1} dx}{\sqrt[n]{(1-x^n)^{n-m}}} = -\frac{1}{n^2} \int_0^1 t^{\mu/n-1} (1-t)^{m/n-1} \ln t dt.$$

The result now follows from the identity

$$\int_0^1 t^{a-1} (1-t)^{b-1} \ln t dt = B(a, b) [\psi(a) - \psi(a+b)].$$