

PROOF OF FORMULA 4.257.3

$$\int_0^\infty \left(\frac{x^p}{q^{2p} + x^{2p}} \right)^r \ln \frac{x}{q} \frac{dx}{q^2 + x^2} = 0$$

Let $x = qt$ to produce

$$\int_0^\infty \left(\frac{x^p}{q^{2p} + x^{2p}} \right)^r \ln \frac{x}{q} \frac{dx}{q^2 + x^2} = q^{-rp-1} \int_0^\infty \left(\frac{t^p}{1 + t^{2p}} \right)^r \frac{\ln t}{1 + t^2} dt.$$

Split the integral over $[0, 1]$ and $[1, \infty)$ and let $s = 1/t$ in the second interval. Then check that this integral becomes minus the one over $[0, 1]$. Thus the total integral vanishes.