

PROOF OF FORMULA 4.261.12

$$\int_0^1 \frac{x^n \ln^2 x}{1-x} dx = 2 \sum_{k=n}^{\infty} \frac{1}{(k+1)^3} = 2 \left(\zeta(3) - \sum_{k=1}^n \frac{1}{k^3} \right)$$

Let $x = e^{-t}$ to obtain

$$\int_0^1 \frac{x^n \ln^2 x}{1-x} dx = \int_0^{\infty} \frac{t^2 e^{-(n+1)t}}{1-e^{-t}} dt.$$

The result now follows from formula **3.411.14**.