PROOF OF FORMULA 4.264.2

$$\int_0^1 \frac{\ln^5 x \, dx}{1-x} = -\frac{8\pi^6}{63}$$

Entry **4.271.4** states that

$$\int_0^1 \frac{\ln^{p-1} x \, dx}{1-x} = (-1)^{p-1}\Gamma(p)\zeta(p).$$

Take $p = 6$ and use $\Gamma(6) = 120$ and $\zeta(6) = \pi^6/945$ to obtain the result.