PROOF OF FORMULA 4.267.1

$$\int_0^1 \frac{1-x}{1+x}\,\frac{dx}{\ln x} = \ln\frac{2}{\pi}$$

The change of variable $t = -\ln x$ yields

$$\int_0^1 \frac{1-x}{1+x}\,\frac{dx}{\ln x} = -\int_0^\infty \frac{e^{-t}-e^{-2t}}{1+e^{-t}}\,\frac{dt}{t}.$$

This is a special case of entry **3.411.28** with $\nu = 1$ and $\mu = 2$. The results follows from there.