

**PROOF OF FORMULA 4.267.9**

$$\int_0^1 \frac{x^{p-1} - x^{q-1}}{\ln x} \frac{dx}{1+x} = \ln \left( \frac{\Gamma(\frac{q}{2})\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2})\Gamma(\frac{q+1}{2})} \right)$$

The change of variables  $x = e^{-t}$  gives

$$\int_0^1 \frac{x^{p-1} - x^{q-1}}{\ln x} \frac{dx}{1+x} = \int_0^\infty \frac{e^{-qt} - e^{-pt}}{1 + e^{-t}} \frac{dt}{t}.$$

This last integral appears in entry **3.411.28**.