PROOF OF FORMULA 4.272.8

$$\int_0^1 \left(\ln \frac{1}{x} \right)^{n-1} \frac{x^{\nu-1}\,dx}{1+x} = (n-1)! \sum_{k=0}^{\infty} \frac{(-1)^k}{(\nu+k)^n}$$

Expand the integrand as a geometric series and let $u = \ln 1/x$ to obtain

$$\begin{aligned} \int_0^1 \left(\ln \frac{1}{x} \right)^{n-1} \frac{x^{\nu-1}\,dx}{1+x} &= \sum_{k=0}^{\infty} (-1)^k \int_0^1 \left(\ln \frac{1}{x} \right)^{n-1} x^{\nu+k-1}\,dx \\ &= \sum_{k=0}^{\infty} (-1)^k \int_0^{\infty} u^{n-1} e^{-(\nu+k-1)u}\,du. \end{aligned}$$

The result now follows from the change of variables $t = (\nu+k)u$.