PROOF OF FORMULA 2.321.2

$$\int x^n e^{ax}\, dx = e^{ax} \left(\sum_{k=0}^{n} \frac{(-1)^k k! \binom{n}{k}}{a^{k+1}} x^{n-k} \right)$$

Formula 2.321.1 states the recurrence

$$I_n(a) = \frac{1}{a} x^n e^{ax} - \frac{n}{a} I_{n-1}(a)$$

for

$$I_m(a) = \int x^n e^{ax} dx.$$

It is required to check that

$$I_m(a) = n! e^{ax} \sum_{k=0}^{n} \frac{(-1)^k x^{n-k}}{(n-k)! a^{k+1}}.$$

Thsi is equivalent to

$$\sum_{k=0}^{n} \frac{(-1)^k x^{n-k}}{(n-k)! a^{k+1}} = \frac{x^n}{an!} - \sum_{k=0}^{n-1} \frac{(-1)^k x^{n-1-k}}{(n-1-k)! a^{k+2}}.$$

This is elementary, the right hand side is simplified using

$$\sum_{k=0}^{n-1} \frac{(-1)^k x^{n-1-k}}{(n-1-k)! a^{k+2}} = \sum_{k=1}^{n} \frac{(-1)^{k+1} x^{n-k}}{(n-k)! a^{k+1}},$$

and the term $x^n/an!$ corresponds to the index $k = 0$.