

PROOF OF FORMULA 2.323

$$\int P_m(x)e^{ax} dx = \frac{e^{ax}}{a} \sum_{k=0}^m \frac{(-1)^k}{a^k} P^{(k)}(x),$$

where P_m is a polynomial of degree m .

Integrate by parts to get

$$\int P_m(x)e^{ax} dx = \frac{1}{a}e^{ax}P_m(x) - \frac{1}{a} \int P_m'(x)e^{ax} dx.$$

Induction on the degree of P gives

$$\begin{aligned} \int P_m(x)e^{ax} dx &= \frac{1}{a}e^{ax}P_m(x) - \frac{e^{ax}}{a^2} \sum_{k=0}^{m-1} \frac{(-1)^k}{a^k} P_m^{(k+1)}(x) \\ &= \frac{e^{ax}}{a} \sum_{k=0}^m \frac{(-1)^k}{a^k} P_m^{(k)}(x), \end{aligned}$$

as claimed.