PROOF OF FORMULA 2.324.2

$$\int \frac{e^{ax}\,dx}{x^m} = -\frac{e^{ax}}{(m-1)!}\sum_{k=1}^{m-1}\frac{a^{k-1}\,(m-k-1)!}{x^{m-k}} + \frac{a^{m-1}}{(m-1)!}\text{Ei}(ax)$$

Formula 2.324.1 states that

$$I_m(a) := \int x^{-m}e^{ax}\,dx$$

satisfies the recurrence

$$I_m(a) = -\frac{e^{ax}}{(m-1)x^{m-1}} + \frac{a}{m-1}I_{m-1}(a).$$

It is direct to check that the right hand side of the formula satisfies the same recurrence. Indeed,

$$-\frac{e^{ax}}{(m-1)x^{m-1}} + \frac{a}{m-1}\left(\frac{-e^{ax}}{(m-2)!}\sum_{k=1}^{m-2}\frac{a^{k-1}(m-k-2)!}{x^{m-1-k}} + \frac{a^{m-2}}{(m-2)!}\text{Ei}(ax)\right).$$

shows that the terms containig the exponential function Ei will match. Morever, the first term in the previous expression corresponds to the case $k=1$ in the formula for the integral. It remains to prove

$$-\frac{e^{ax}}{(m-1)!}\sum_{k=2}^{m-1}\frac{a^{k-1}\,(m-k-1)!}{x^{m-k}} = \frac{a}{m-1}\left(\frac{-e^{ax}}{(m-2)!}\sum_{k=1}^{m-2}\frac{a^{k-1}(m-k-2)!}{x^{m-1-k}}\right).$$

This is done by shifting the index of the sum on the right.