

PROOF OF FORMULA 3.192.3

$$\int_0^1 \frac{(1-x)^p dx}{x^{p+1}} = -\frac{\pi}{\sin(\pi p)}$$

The integral representation

$$B(a, b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx$$

shows that the claimed formula is

$$B(p+1, -p) = \Gamma(1+p)\Gamma(-p).$$

The result now follows from $\Gamma(z)\Gamma(1-z) = \pi/\sin \pi z$, with $z = 1+p$.