

PROOF OF FORMULA 3.192.4

$$\int_1^{\infty} (x-1)^{p-1/2} \frac{dx}{x} = \frac{\pi}{\cos \pi p}$$

Let $t = 1/x$ to obtain

$$\begin{aligned} \int_1^{\infty} (x-1)^{p-1/2} \frac{dx}{x} &= \int_0^1 t^{-1/2-p} (1-t)^{p-1/2} dt \\ &= B\left(\frac{1}{2}-p, \frac{1}{2}+p\right). \end{aligned}$$

The result now follows from the identity

$$\begin{aligned} B\left(\frac{1}{2}-p, \frac{1}{2}+p\right) &= \Gamma\left(\frac{1}{2}-p\right) \Gamma\left(\frac{1}{2}+p\right) \\ &= \frac{\pi}{\cos \pi p}. \end{aligned}$$