PROOF OF FORMULA 3.194.2

$$\int_a^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = \frac{a^{\mu-\nu}}{b^\nu(\nu-\mu)} {}_2F_1\left[\nu, \nu-\mu; \nu-\mu+1; -1/ab\right]$$

Let $t = a/x$ to obtain

$$\int_a^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = \frac{a^{\mu-\nu}}{b^\nu} \int_0^1 t^{\nu-\mu-1}(1+t/ab)^\nu\,dt.$$

The integral representation of the hypergeometric function

$${}_2F_1[\alpha,\beta;\gamma;z] = \frac{1}{B(\beta,\gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^{-\nu}\,dt$$

is used with $\alpha = \nu$, $\beta = \nu-\mu$, $\gamma = \nu-\mu+1$ and $z = -1/ab$ to produce the resul. The value $B(c,1) = 1/c$ is used in the simplification of the answer.