PROOF OF FORMULA 3.194.8

$$\int_0^1 \frac{x^{n-1}\,dx}{(1+x)^m} = \sum_{k=0}^\infty \binom{m-n-1}{k} \frac{(-1)^k}{2^{n+k}(n+k)}$$

Let $t = x/(1+x)$ to obtain

$$\int_0^1 \frac{x^{n-1}\,dx}{(1+x)^m} = \int_0^{1/2} \frac{t^{n-1}\,dt}{(1-t)^{n-m+1}}.$$

The binomial theorem gives

$$(1-t)^{m-n-1} = \sum_{k=0}^\infty (-1)^k \binom{m-n-1}{k} t^k,$$

and thus

$$\int_0^1 \frac{x^{n-1}\,dx}{(1+x)^m} = \sum_{k=0}^\infty (-1)^k \binom{m-n-1}{k} \int_0^{1/2} t^{n+k-1}\,dt.$$

The result follows by integrating term by term.