PROOF OF FORMULA 3.196.1

$$\int_0^a (x+b)^\nu (a-x)^{\mu-1}\, dx = \frac{b^\nu a^\mu}{\mu} {}_2F_1\left[1, -\nu, 1+\mu; -a/b\right]$$

Let $x = at$ to obtain

$$\int_0^a (x+b)^\nu (a-x)^{\mu-1}\, dx = b^\nu a^\mu \int_0^1 (1-t)^{\mu-1}(1+at/b)^\nu\, dt.$$

The result now follows from the integral representation for the hypergeometric function

$${}_2F_1[\alpha, \beta; \gamma; z] = \frac{1}{B(\beta, \gamma-\beta)} \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-tz)^\alpha\, dt.$$