

PROOF OF FORMULA 3.197.2

$$\int_a^\infty x^{-\lambda}(x+b)^\nu(x-a)^{\mu-1} dx = a^{-\lambda}(a+b)^{\mu+\nu} B(\mu, \lambda-\mu-\nu) {}_2F_1[\lambda, \mu; \lambda-\nu+\rho; -b/a]$$

Let $t = x - a$ to obtain

$$\int_0^\infty (t+a)^{-\lambda}(t+a+b)^\nu t^{\mu-1} dt.$$

Now employ 3.197.1 written in the form

$$\int_0^\infty x^{\alpha-1}(x+B)^{-\beta}(x+C)^{-\gamma} dx = B^{-\beta}C^{\alpha-\gamma} B(\alpha, \beta-\alpha+\gamma) {}_2F_1[\beta, \alpha; \beta+\gamma; 1-C/B],$$

with $\alpha = \mu$, $\beta = \lambda$, $\gamma = -\nu$ and $B = a$, $C = a + b$, to get the result.