PROOF OF FORMULA 3.197.4

$$\int_0^1 x^{\mu-1}(1-x)^{\nu-1}(1+ax)^{-\mu-\nu}\,dx = (1+a)^{-\mu}B(\mu,\nu)$$

The integral representation

$$_2F_1\left[\alpha,\beta;\gamma;z\right] = \frac{1}{B(\beta,\gamma-\beta)}\int_0^1 x^{\beta-1}(1-x)^{\gamma-\beta-1}(1-zx)^{-\alpha}dx,$$

gives

$$\int_0^1 x^{\mu-1}(1-x)^{\nu-1}(1+ax)^{-\mu-\nu}\,dx = B(\mu,\nu)\,_2F_1\left[\mu+\nu,\mu;\mu+\nu;-a\right].$$

The result follows from the identity (9.121.1):

$$_2F_1\left[-r,\beta;\beta;-a\right] = \,_2F_1\left[\beta,-r;\beta;-a\right] = (1+a)^r.$$