PROOF OF FORMULA 3.197.9

$$\int_0^\infty x^{\lambda-1}(1+x)^{-\mu+\nu}(x+b)^{-\nu}\,dx = B(\mu-\lambda,\lambda)\,{}_2F_1\,[\nu,\mu-\lambda;\mu;1-b]$$

The integral is

$$b^{-\nu}\int_0^\infty x^{\lambda-1}(1+x)^{-\mu+\nu}(1+x/b)^{-\nu}\,dx = b^{-\nu}B(\lambda,\mu-\lambda)\,{}_2F_1[\nu,\lambda;\mu;1-1/b]$$

using 3.197.5. The formula is now simplified using

$${}_2F_1[\alpha,\beta;\gamma;z] = (1-z)^{-\alpha}\,{}_2F_1[\alpha,\gamma-\beta;\gamma;z/(z-1)]$$

to obtain the result.