FORMULA 3.217

$$\int_0^\infty \left[\frac{b^p x^{p-1}}{(1+bx)^p} - \frac{(1+bx)^{p-1}}{b^{p-1}x^p} \right] dx = \frac{\pi}{\tan \pi p}$$

The change of variables $t = bx$ shows that the integral is independent of the parameter b. Therefore

$$\int_0^\infty \left[\frac{b^p x^{p-1}}{(1+bx)^p} - \frac{(1+bx)^{p-1}}{b^{p-1}x^p} \right] dx = \int_0^\infty \left[\frac{t^{p-1}}{(1+t)^p} - \frac{(1+t)^{p-1}}{t^p} \right] dt.$$

The change of variables $y = t/(1+t)$ gives

$$\int_0^\infty \left[\frac{t^{p-1}}{(1+t)^p} - \frac{(1+t)^{p-1}}{t^p} \right] dt = \int_0^1 \frac{y^{p-1} - y^{-p}}{1-y} dy.$$

This is evaluated in 3.231.1 as $\pi \cot(\pi p)$.