PROOF OF FORMULA 3.218

$$\int_0^\infty \frac{x^{2p-1} - (a+x)^{2p-1}}{(a+x)^p x^p} \, dx = \frac{\pi}{\tan \pi p}$$

The change of variables $x = at$ shows that the integral is independent of a:

$$\int_0^\infty \frac{x^{2p-1} - (a+x)^{2p-1}}{(a+x)^p x^p} \, dx = \int_0^\infty \frac{t^{2p-1} - (1+t)^{2p-1}}{(1+t)^p \, t^p} \, dt$$

Then,

$$\int_0^\infty \frac{t^{2p-1} - (1+t)^{2p-1}}{(1+t)^p \, t^p} \, dt = \int_0^\infty \left(\frac{t^{p-1}}{(1+t)^p} - \frac{(1+t)^{p-1}}{t^p} \right) \, dt.$$

This is evaluated as $\pi \cot(\pi p)$ in 3.217.