PROOF OF FORMULA 3.223.1

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(x+a)(x+b)} = -\frac{\pi}{\sin\pi\mu}\frac{b^{\mu-1}-a^{\mu-1}}{b-a}$$

The partial fraction decomposition

$$\frac{1}{(x+a)(x+b)} = \frac{1}{a-b}\left(\frac{1}{x+b} - \frac{1}{x+a}\right)$$

gives

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(x+a)(x+b)} = \frac{1}{a-b}\int_0^\infty \frac{x^{\mu-1}\,dx}{x+b} - \frac{1}{a-b}\int_0^\infty \frac{x^{\mu-1}\,dx}{x+a}.$$

Let $t = bx$ in the first integral and $t = ax$ in the second one and then use

$$\int_0^\infty \frac{t^{\mu-1}\,dt}{1+t} = \frac{\pi}{\sin\pi\mu}$$

to obtain the result.