PROOF OF FORMULA 3.226.2

$$\int_0^1 \frac{x^{n-1/2}\,dx}{\sqrt{1-x}} = \frac{(2n-1)!!}{(2n)!!}\,\pi.$$

The integral representation

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx$$

gives

$$
\begin{aligned}
\int_0^1 \frac{x^{n-1/2}\,dx}{\sqrt{1-x}} &= B\left(n+\tfrac{1}{2},\tfrac{1}{2}\right) \\
&= \frac{\Gamma(n+1/2)\,\Gamma(1/2)}{\Gamma(n+1)}.
\end{aligned}
$$

The result follows from

$$\Gamma\left(m+\tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{m-1}}(2m-1)!!$$