

PROOF OF FORMULA 3.227.1

$$\int_0^\infty \frac{x^{\nu-1} (x+b)^{1-\mu}}{x+a} dx = b^{1-\mu} a^{\nu-1} B(\nu, \mu - \nu) {}_2F_1 [\mu - 1, \nu; \mu; 1 - a/b]$$

The integral is

$$\int_0^\infty \frac{x^{\nu-1} (x+b)^{1-\mu}}{x+a} dx = \int_0^\infty x^{\nu-1} (x+b)^{1-\mu} (x+a)^{-1} dx.$$

Formula 3.197.1 states that

$$\int_0^\infty x^{\nu-1} (x+b)^{-\mu} (x+a)^{-\rho} dx = b^{-\mu} a^{\nu-\rho} B(\nu, \mu - \nu + \rho) {}_2F_1 [\mu, \nu; \mu + \rho; 1 - a/b].$$

Replace μ by $\mu - 1$ and ρ by 1 to get the result requested here.