PROOF OF FORMULA 3.229

$$\int_0^1 \frac{x^{\mu-1}\,dx}{(1-x)^\mu(1+ax)(1+bx)} = \frac{\pi}{(a-b)\sin\pi\mu}\left[\frac{a}{(1+a)^\mu} - \frac{b}{(1+b)^\mu}\right]$$

The partial fraction expansion

$$\frac{1}{(1+ax)(1+bx)} = \frac{a}{a-b}\frac{1}{1+ax} - \frac{b}{a-b}\frac{1}{1+bx}$$

gives

$$\int_0^1 \frac{x^{\mu-1}\,dx}{(1-x)^\mu(1+ax)(1+bx)} = \frac{a}{a-b}\int_0^1 \frac{x^{\mu-1}\,dx}{(1-x)^\mu(1+ax)} - \frac{b}{a-b}\int_0^1 \frac{x^{\mu-1}\,dx}{(1-x)^\mu(1+bx)}.$$

The result now follows from entry 3.197.10 which states

$$\int_0^1 \frac{x^{q-1}\,dx}{(1-x)^q(1+px)} = \frac{\pi}{\sin(\pi q)\,(1+p)^q}.$$