PROOF OF FORMULA 3.238.2

$$\int_{-\infty}^{\infty} \frac{|x|^{\nu-1}}{x-u} \text{ sign } x\, dx = \pi \tan\left(\frac{\pi\nu}{2}\right) |u|^{\nu-1}$$

Let $x = tu$ to obtain

$$\int_{-\infty}^{\infty} \frac{|x|^{\nu-1}}{x-u} \text{ sign } x\, dx = |u|^{\nu-1} \int_{-\infty}^{\infty} \frac{|t|^{\nu-1}}{1-t} \text{sign t } dt.$$

Now split the integral to obtain

$$\int_{-\infty}^{\infty} \frac{|t|^{\nu-1}}{1-t} \text{sign } t\, dt \quad = \quad \int_0^{\infty} \frac{\sigma^{\nu-1} d\sigma}{1+\sigma} - \int_0^{\infty} \frac{\sigma^{\nu-1} d\sigma}{1-\sigma}.$$

These integrals are evaluated in 3.241.3 and 3.241.2 respectively. It follows that

$$\int_{-\infty}^{\infty} \frac{|t|^{\nu-1}}{1-t} \text{ sign } t\, dt = \frac{\pi}{\sin \pi\nu} - \frac{\pi}{\tan \pi\nu}.$$

This simplifies to $\tan\left(\frac{\pi\nu}{2}\right)$.