PROOF OF FORMULA 3.241.3

$$\int_0^\infty \frac{x^{p-1}\,dx}{1-x^q} = \frac{\pi}{q}\cot\left(\frac{\pi p}{q}\right)$$

Let $t = x^q$ to obtain

$$\int_0^\infty \frac{x^{p-1}\,dx}{1-x^q} = \frac{1}{q}\int_0^\infty \frac{t^a\,dt}{1-t}$$

with $a = p/q$. Split this integral at $t = 1$ and change t by $1/t$ in the range $t \geq 1$ to produce

$$\int_0^\infty \frac{x^{p-1}\,dx}{1-x^q} = \frac{1}{q}\int_0^1 \frac{t^{a-1}-t^{-a}}{1-t}\,dt.$$

This last integral is evaluated as $\pi\cot\pi a$ in entry 3.231.