PROOF OF FORMULA 3.241.5

$$\int_0^\infty \frac{x^{p-1}\,dx}{(1+x^q)^2} = \frac{q-p}{q^2}\frac{\pi}{\sin(\pi p/q)}$$

Let $t = x^q$ to obtain

$$\int_0^\infty \frac{x^{p-1}\,dx}{(1+x^q)^2} = \frac{1}{q}\int_0^\infty \frac{t^{p/q-1}\,dt}{(1+t)^2}.$$

The integral representation

$$B(a,b) = \int_0^\infty \frac{t^{a-1}\,dt}{(1+t)^{a+b}},$$

shows that the requested evaluation is

$$\frac{1}{q}B\left(\frac{p}{q}, 2-\frac{p}{q}\right) = \frac{1}{q}\Gamma(p/q)\Gamma(2-p/q).$$

The result is simplified using $\Gamma(a+1) = a\Gamma(a)$ and $\Gamma(a)\Gamma(1-a) = \pi/\sin(\pi a)$.