PROOF OF FORMULA 3.244.2

$$\int_0^1 \frac{x^{p-1} - x^{q-p-1}}{1 - x^q}\, dx = \frac{\pi}{q} \cot\left(\frac{\pi p}{q}\right)$$

Let $t = x^q$ to obtain

$$\int_0^1 \frac{x^{p-1} - x^{q-p-1}}{1 - x^q}\, dx = \frac{1}{q} \int_0^1 \frac{t^{p/q-1} - t^{-p/q}}{1 - t}\, dt.$$

The result now follows from the formula 3.231.1:

$$\int_0^1 \frac{x^{p-1} - x^{-p}}{1 - x}\, dx = \pi \cot \pi p.$$