PROOF OF FORMULA 3.244.3

$$\int_0^1 \frac{x^{\nu-1}-x^{\mu-1}}{1-x^{\nu}}\,dx = \frac{1}{\nu}\left[\gamma+\psi\left(\frac{\mu}{\nu}\right)\right]$$

Let $t = x^{\nu}$ to obtain

$$\int_0^1 \frac{x^{\nu-1}-x^{\mu-1}}{1-x^{\nu}}\,dx = \frac{1}{\nu}\int_0^1 \frac{1-t^{\mu/\nu-1}}{1-t}\,dt.$$

The result now follows from the representation

$$\psi(a) = \int_0^1 \frac{t^{a-1}-1}{t-1}\,dt - \gamma,$$

given in 8.361.7.