PROOF OF FORMULA 3.245

$$\int_0^\infty \left[x^{\nu-\mu} - x^\nu (1+x)^{-\mu} \right] \, dx = \frac{\nu}{\nu-\mu+1} B(\nu, \mu-\nu)$$

The change of variables $x = \tan^2 \alpha$ gives

$$\begin{aligned}\int_0^\infty \left[x^{\nu-\mu} - x^\nu (1+x)^{-\mu} \right] \, dx &= 2 \int_0^{\pi/2} (\sin\alpha)^{2(\nu-\mu+1)-1} (\cos\alpha)^{2(-\nu+\mu-1)-1} \, d\alpha \\ &- 2 \int_0^{\pi/2} (\sin\alpha)^{2(\nu+1)-1} (\cos\alpha)^{2(-\nu+\mu-1)-1} \, d\alpha.\end{aligned}$$

The integral representation 8.380.2

$$B(u,v) = 2 \int_0^{\pi/2} \sin^{2u-1} \alpha \, \cos^{2v-1} \alpha \, d\alpha$$

produces

$$\int_0^\infty \left[x^{\nu-\mu} - x^\nu (1+x)^{-\mu} \right] \, dx = B(\nu-\mu+1, -\nu+\mu-1) - B(\nu+1, -\nu+\mu-1)$$

and the result reduces to the stated formula via

$$B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}.$$