PROOF OF FORMULA 3.247.2

$$\int_0^\infty \frac{(1-x^p)x^{\nu-1}}{1-x^{np}}\,dx = \frac{\pi}{np}\sin\left(\frac{\pi}{n}\right)\operatorname{cosec}\left(\frac{(p+\nu)\pi}{np}\right)\operatorname{cosec}\left(\frac{\nu\pi}{np}\right)$$

Split the integral into the two intervals $[0,1]$ and $[1,\infty)$. Let $s = 1/x$ in the second part to obtain

$$\int_0^\infty \frac{(1-x^p)x^{\nu-1}}{1-x^{np}}\,dx = \int_0^1 \frac{(1-x^p)x^{\nu-1}}{1-x^{np}}\,dx + \int_0^1 \frac{(1-x^p)x^{np-\nu-1-p}}{1-x^{np}}\,dx.$$

The change of variables $x = t^{np}$ yields

$$\int_0^\infty \frac{(1-x^p)x^{\nu-1}}{1-x^{np}}\,dx = \frac{1}{np}\int_0^1 \frac{(t^{b-1}-t^{-b})-(t^{c-1}-t^{-c})}{1-t}\,dt$$

where $b = \nu/np$ and $c = 1/n + \nu/np$. Now employ entry 3.231.1

$$\int_0^1 \frac{t^{p-1}-t^{-p}}{1-t}\,dt = \pi\cot\pi p$$

to obtain the result.