

PROOF OF FORMULA 3.251.2

$$\int_0^{\infty} x^{\mu-1}(1+x^2)^{\nu-1} dx = \frac{1}{2} B\left(\frac{\mu}{2}, 1-\nu-\frac{\mu}{2}\right)$$

Let $t = x^2$ to obtain

$$\int_0^{\infty} x^{\mu-1}(1+x^2)^{\nu-1} dx = \frac{1}{2} \int_0^{\infty} t^{\mu/2-1}(1+t)^{\nu-1} dt.$$

The result now follows from the representation for the beta function

$$B(a, b) = \int_0^{\infty} \frac{t^{a-1} dt}{(1+t)^{a+b}},$$

with $a = \mu/2$ and $b = 1 - \nu - \mu/2$.