PROOF OF FORMULA 3.251.3

$$\int_1^\infty x^{\mu-1}(x^p-1)^{\nu-1}\,dx = \frac{1}{p}B\left(\nu, 1-\nu-\frac{\nu}{p}\right)$$

Let $t = 1/x$ to obtain

$$\int_1^\infty x^{\mu-1}(x^p-1)^{\nu-1}\,dx = \int_0^1 t^{-1-\mu+p-\nu p}(1-t^p)^{\nu-1}\,dt.$$

The change of variables $s = t^p$ gives

$$\int_1^\infty x^{\mu-1}(x^p-1)^{\nu-1}\,dx = \frac{1}{p}\int_0^1 s^{-\nu-\mu/p}(1-s)^{\nu-1}\,ds,$$

and the result follows from the integral representation of the beta function

$$B(a,b) = \int_0^1 s^{a-1}(1-s)^{b-1}\,ds.$$