PROOF OF FORMULA 3.252.1

$$\int_0^\infty \frac{dx}{(ax^2+2bx+c)^n} = \frac{(-1)^{n-1}}{(n-1)!}\frac{\partial^{n-1}}{\partial c^{n-1}}\left[\frac{\text{ArcCot}\left(b/\sqrt{ac-b^2}\right)}{\sqrt{ac-b^2}}\right]$$

Start with the case $n=1$:

$$\int_0^\infty \frac{dx}{ax^2+2bx+c} = \frac{1}{a}\int_0^\infty \frac{dx}{x^2+2bx/a+c/a}.$$

Complete the square to get

$$x^2+\frac{2bx}{a}+\frac{c}{a} = \left(x+\frac{b}{a}\right)^2+\frac{ac-b^2}{a^2}.$$

The change of variables $v = \frac{a}{\sqrt{ac-b^2}}\left(x+\frac{b}{a}\right)$ gives

$$\int_0^\infty \frac{dx}{ax^2+2bx+c} = \frac{1}{\sqrt{ac-b^2}}\int_\alpha^\infty \frac{dv}{v^2+1},$$

where $\alpha = b/\sqrt{ac-b^2}$. This gives the result for $n=1$.

To conclude the case $n>1$, differentiate with respect to the parameter c.