PROOF OF FORMULA 3.252.3

$$\int_0^\infty \frac{dx}{(ax^2+2bx+c)^{n+3/2}} = \frac{(-2)^n}{(2n+1)!!} \frac{\partial^n}{\partial c^n} \left[\frac{1}{\sqrt{c}\,(\sqrt{ac}+b)} \right]$$

The case $n = 0$ is solved by the change of variables $u = a(x+b/a)/(ac-b^2)^{1/2}$ to obtain

$$\int_0^\infty \frac{dx}{(ax^2+2bx+c)^{3/2}} = \frac{\sqrt{a}}{ac-b^2} \int_{a^*}^\infty \frac{du}{(u^2+1)^{3/2}},$$

where $a^* = b/\sqrt{ac-b^2}$. The change of variables $u = \tan\phi$ yields

$$\int_0^\infty \frac{dx}{(ax^2+2bx+c)^{3/2}} = \frac{1}{\sqrt{c}\,(\sqrt{ac}+b)}.$$

The formula for $n > 0$ comes by differentiation with respect to c.