PROOF OF FORMULA 3.267.3

$$\int_0^1 \frac{x^{3n-2}\,dx}{\sqrt[3]{1-x^3}} = \frac{\Gamma(n-\frac{1}{3})\Gamma\left(\frac{2}{3}\right)}{3\Gamma\left(n+\frac{1}{3}\right)}$$

Let $t = x^3$ to obtain

$$\int_0^1 \frac{x^{3n-2}\,dx}{\sqrt[3]{1-x^3}} = \frac{1}{3}\int_0^1 \frac{t^{n-\frac{4}{3}}\,dt}{(1-t)^{1/3}}.$$

The integral representation

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt$$

gives the last integral as

$$B\left(n-\tfrac{1}{3},\tfrac{2}{3}\right).$$

The result is simplified using

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$