

**PROOF OF FORMULA 3.268.1**

$$\int_0^1 \left( \frac{1}{1-x} - \frac{px^{p-1}}{1-x^p} \right) dx = \ln p$$

Define

$$\begin{aligned} I(\epsilon) &:= \int_0^{1-\epsilon} \left( \frac{1}{1-x} - \frac{px^{p-1}}{1-x^p} \right) dx \\ &= -\ln \epsilon + \ln(1 - (1-\epsilon)^p) \\ &= \ln \left[ \frac{1 - (1-\epsilon)^p}{\epsilon} \right]. \end{aligned}$$

The result follows by letting  $\epsilon \rightarrow 0$ .