PROOF OF FORMULA 3.269.1

$$\int_0^1 x \cdot \frac{x^p - x^{-p}}{1-x^2}\, dx = \frac{\pi}{2} \cot\left(\frac{\pi p}{2}\right) - \frac{1}{p}$$

Let $t = x^2$ to obtain

$$\int_0^1 \frac{x^p - x^{-p}}{1-x^2}\, dx = \frac{1}{2} \int_0^1 \frac{t^{p/2} - t^{-p/2}}{1-t}\, dt.$$

Formula 3.231.5 states that

$$\int_0^1 \frac{x^{\mu-1} - x^{\nu-1}}{1-x}\, dx = \psi(\nu) - \psi(\mu),$$

so that,

$$\int_0^1 \frac{x^p - x^{-p}}{1-x^2}\, dx = \frac{1}{2}\left[\psi(1-p/2) - \psi(1+p/2)\right].$$

The result follows from

$$\psi(x+1) = \psi(x) + 1/x \text{ and } \psi(1-x) = \psi(x) + \pi\cot(\pi x).$$