PROOF OF FORMULA 3.311.13

$$\int_0^\infty \frac{e^{-px}+e^{-qx}}{1+e^{-(p+q)x}}\,dx = \frac{\pi}{p+q}\operatorname{cosec}\left(\frac{\pi p}{p+q}\right)$$

Let $t = e^{-(p+q)x}$ to get

$$\int_0^\infty \frac{e^{-px}+e^{-qx}}{1+e^{-(p+q)x}}\,dx = \frac{1}{p+q}\int_0^1 \frac{t^{r-1}+t^{-r}}{1+t}\,dt,$$

with $r = p/(p+q)$.

Formula 3.231.2 gives $\pi\operatorname{cosec}(\pi r)$ as the value of this last integral.