PROOF OF FORMULA 3.311.3

$$\int_{-\infty}^{\infty} \frac{e^{-px}\,dx}{1+e^{-qx}} = \frac{\pi}{q\sin(\pi p/q)}$$

Let $t = e^{-x}$ to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-px}\,dx}{1+e^{-qx}} = \int_0^{\infty} \frac{t^{p-1}\,dt}{1+t^q}.$$

The change of variables $u = t^q$ gives

$$\int_0^{\infty} \frac{t^{p-1}\,dt}{1+t^q} = \frac{1}{q}\int_0^{\infty} \frac{u^{p/q-1}\,du}{1+u}.$$

The integral representation

$$B(a,b) = \int_0^{\infty} \frac{u^{a-1}\,du}{(1+u)^{a+b}},$$

shows that

$$\frac{1}{q}\int_0^{\infty} \frac{u^{p/q-1}\,du}{1+u} = \frac{1}{q}B\left(\frac{p}{q}, 1-\frac{p}{q}\right).$$

The answer is simplified using

$$B(a,1-a) = \Gamma(a)\Gamma(1-a) = \frac{\pi}{\sin \pi a}.$$