PROOF OF FORMULA 3.313.1

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{1-e^{-x}} = \pi \cot \pi\mu$$

The change of variables $t = e^{-x}$ yields

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{1-e^{-x}} = \int_{0}^{\infty} \frac{t^{\mu-1}\,dt}{1-t}.$$

This is the special case $a = -1$ in formula 3.222.2:

$$\int_{0}^{\infty} \frac{x^{\mu-1}\,dx}{x+a} = -\pi \cot \pi\mu (-a)^{\mu-1} \text{ for } a < 0.$$