

PROOF OF FORMULA 3.313.2

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{(1 + e^{-x})^\nu} = B(\mu, \nu - \mu)$$

Let $t = e^{-x}$ to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{(1 + e^{-x})^\nu} = \int_0^{\infty} \frac{t^{\mu-1} dt}{(1+t)^\nu}$$

The result follows from the integral representation

$$B(a, b) = \int_0^{\infty} \frac{t^{a-1} dt}{(1+t)^{a+b}},$$

with $a = \mu$ and $b = \nu - \mu$.